QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION

MATHEMATICS 142 $2\frac{1}{2}$ HOURS SPRING 2015

INSTRUCTIONS: ANSWER ALL QUESTIONS SHOW ALL WORK

1. Evaluate the following integrals without the use of a calculator.

(a) $\displaystyle\int \frac{z^2}{\sqrt{1+z^3}}\,dz$

(b) $\displaystyle\int_0^5 \sqrt{25-x^2}\,dx$

(c) $\displaystyle\int \frac{4+u^2}{u^3}\,du$

(d) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

2. Using the definition of a definite integral as the limit of the Riemann sum, evaluate

$$\int_0^2 (x^2+2x)dx.$$

Note: $\displaystyle\sum_{k=1}^{n} k = \frac{(n)(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

3. If $F(x) = \int_1^x f'(t)dt$ where $f(t) = \int_1^{t^2} \left(u+2u^2\right) du$, find $F'(3)$.

4. In each case, calculate the derivative of the given function.

(a) $f(x) = \dfrac{(x^3+1)^3 \sin^2 x}{\sqrt[3]{x}}$

(b) $g(x) = \cos^{-1}(e^x)$

(c) $h(x) = (\ln x)^{\cos(x)}$

(d) $k(x) = \dfrac{\tan^{-1}(3x)}{\ln(3x^2+6x)}$

5. Show that a sphere of radius r has a volume of $\frac{4}{3}\pi r^3$.
(Hint: Rotate the semicircle $y = \sqrt{r^2-x^2}$ about an appropriate axis).

6. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

7. (a) Show that $f(x) = \frac{4x-1}{2x+3}$ has an inverse $f^{-1}(x)$ and then (b) Calculate the derivative of $f^{-1}(x)$ in two ways:

(i) by differentiating the expression for $f^{-1}(x)$,

(ii) by using the general formula for the derivative of an inverse function.

8. Find the arc length function for the curve $y = f(x) = x^2 - \frac{1}{8}\ln(x)$ taking $P_0(1,1)$ as the starting point. Find the arc length along the curve from $(1,1)$ to $(4, f(4))$.

9. Suppose that the growth rate of bacteria in a dish is proportional to the population of bacteria at any time t, where t is in hours (That is, $\frac{dP}{dt} = kP$). Let $P = 250,000$ at $t = 0$ and $P = 422,000$ at $t = 12$. Determine k.

10. Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = \frac{e^{2x}(1+y^3)}{y^2}, \quad y(1) = 0$$